Millikan's Experiment

CARRY OUT MILLIKAN'S EXPERIMENT TO CONFIRM THE VALUE OF THE ELEMENTARY CHARGE WITH THE HELP OF CHARGED OIL DROPS

- **Produce and select suitable oil drops and observe them in an electric field.**
- **Measure the speed with which they rise in the electric field and descend without it.**
- **Confirm the value of the elementary charge.**

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Fig. 1: Millikan's Apparatus

GENERAL PRINCIPLES

Between the years 1910 and 1913, *Robert Andrews Millikan* managed to measure the elementary electric charge to an unprecedented accuracy and thereby confirmed the quantum nature of charge. He was awarded the Nobel Prize in physics for his work. The experiment which now bears his name is based on measuring the quantity of charge carried by charged drops of oil, which are able to rise through the air under the influence of an electric field from a plate capacitor and descend when the field is absent. The value he obtained for the elementary charge $e = (1.592 \pm 0.003) \cdot 10^{-19}$ C differs by only 0.6% from the accepted modern value.

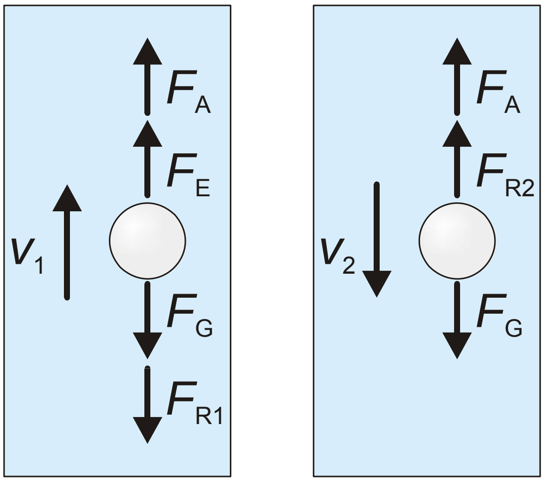

Fig. 2: Schematic of forces acting when an oil drop is rising in the presence of an electric field (left) and descending in the absence of a field (right).

The forces which act on a droplet of oil (which we shall assume to be spherical) situated in the electric field of a plate capacitor (Fig. 2) are:

the force of gravity,

$$(1)\quad F_G = m_2 \cdot g = \frac{4}{3} \cdot \pi \cdot r_0^3 \cdot \rho_2 \cdot g \ ,$$

m_2: Mass of oil drop, r_0: Radius of oil drop, ρ_2: Density of oil, g: Acceleration due to gravity

the drop's buoyancy in air,

$$(2)\quad F_A = \frac{4}{3} \cdot \pi \cdot r_0^3 \cdot \rho_1 \cdot g \ ,$$

ρ_1: Density of air

the force exerted by the electric field E,

$$(3)\quad F_E = q_0 \cdot E = \frac{q_0 \cdot U}{d} \ ,$$

q_0: Charge on oil drop, U: Voltage between the plates of the capacitor, d: Separation of the capacitor plates

and Stokes' force of friction

$$(4)\quad F_{R1,2} = 6 \cdot \pi \cdot \eta \cdot r_0 \cdot v_{1,2} \ .$$

η: Viscosity of air, v_1: Speed of ascent, v_2: Speed of descent

When an oil drop rises in an electric field (Fig. 2), the equilibrium equation involves the following forces:

$$(5)\quad F_G + F_{R1} = F_E + F_A$$

During descent the equation is as follows:

$$(6)\quad F_G = F_{R2} + F_A \ .$$

This means we can find the radius of the drop and its charge:

$$(7)\quad r_0 = \sqrt{\frac{9}{2} \cdot \frac{\eta \cdot v_2}{(\rho_2 - \rho_1) \cdot g}}$$

and

$$(8)\quad q_0 = \frac{6 \cdot \pi \cdot \eta \cdot d \cdot (v_1 + v_2)}{U} \cdot r_0 \ .$$

Very small radii r_0 are of the same order of magnitude as the mean free path of air molecules. This means a correction needs to be made to the Stokes' friction. The corrected radius r and charge q are then given by the following:

$$(9)\quad r = \sqrt{r_0^2 + \frac{A^2}{4}} - \frac{A}{2} \quad \text{where} \quad A = \frac{b}{p}$$

b = 82 µm·hPa = constant, p: air pressure

$$(10)\quad q = q_0 \cdot \left(1 + \frac{A}{r}\right)^{-1,5} .$$

LIST OF EQUIPMENT

1 Millikan's Apparatus @230V 1018884 (U207001-230)
or
1 Millikan's Apparatus @115V 1018882 (U207001-115)

SET-UP

- Set up Millikan's oil drop apparatus on a level surface.
- Turn the vertical adjustment knob clockwise as far as it will go (see Fig. 3).
- Move the measuring microscope to the end of the stand rod on the base unit and secure it in place by means of the knurled screw at the bottom.
- Use the focussing equipment to move the measuring microscope forward as far as it will go and roughly align it in the observation window in the experiment chamber with the help of the vertical adjustment knob.
- Open the cover of the experiment chamber, place the spirit level on the uppermost plate of the plate capacitor and adjust the apparatus to its optimum horizontal alignment with the help of the adjustable feet.

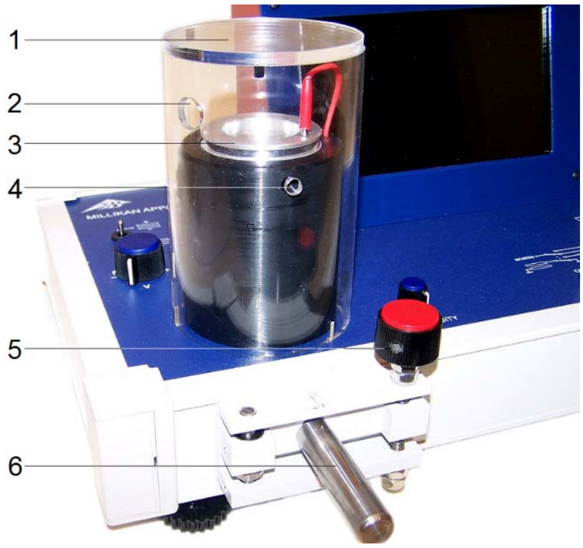

Fig. 3 Experiment chamber: 1 Cover, 2 Hole for oil atomiser, 3 Top capacitor plate, 4 Observation window, 5 Vertical adjustment knob for head of microscope, 6 Stand rod for measuring microscope

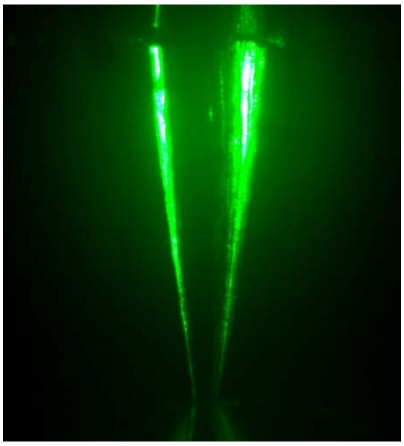

Fig.4 View through the measuring microscope with the adjustment needle in focus

- Insert the adjustment needle in the top plate of the plate capacitor and focus the microscope on the needle (see Fig. 4). Select a suitable light intensity and re-adjust the height of the measuring microscope by means of the vertical adjustment knob.
- Remove the needle and close the experiment chamber.
- Fill the oil atomiser to about halfway with the oil for Millikan's apparatus and carefully slot it into the hole in the experiment chamber.

EXPERIMENT PROCEDURE

Starting the display and control unit

- Connect Millikan's apparatus to the mains via its plug-in power supply.

The display and control unit will be ready to use as soon as Millikan's apparatus has been connected up via the plug-in supply.

- Click the "Select" button ("Wählen") to access the language menu.
- Select the required language by pressing the corresponding button and then click "Enter" to confirm. This automatically returns you to the main menu.
- Click the "Next" button in the main menu to access the measurement menu.

Optimisation of light intensity

- Look through the microscope at the measurement cell (the space between the capacitor plates) and set up a suitable light intensity. Adjust the light intensity during the measurement as needed.

Production, selection and observation of charged oil droplets

- Give a single hard squeeze on the rubber bulb to produce some oil drops and spray them into the measurement cell.

Note:

Once an oil drop has thus acquired, by coincidence, a given charge, there is no longer any external influence which can change it. The Millikan apparatus therefore has no need of any radioactive source. Oil drops are introduced into the experiment chamber, and in particular the measurement cell, from above, just as in Millikan's original experiment.

- Wait till suitable oil droplets appear in the cell. This may take a few seconds.
- Select one of the oil droplets you can see descending slowly (about 0.025 – 0.1 mm/s).
- Adjust the focus of the microscope if necessary.

General notes:

The object is to produce a small number of individual oil droplets, not a large bright cloud of them since you only need to select one droplet. Squeezing the bulb more than once will mean that there are too many droplets in the measurement cell, particularly in the area between the microscope and where it is focussed. Drops there obstruct observation of droplets which are in focus.

A suitable droplet will appear as a brightly lit point in the focus of the measuring microscope.

If too much oil has entered the measurement cell, it will need to be cleaned. If no droplets appear in the measurement cell, even after the bulb has been squeezed multiple times, it may be that the opening in the top capacitor plate has blocked up. Then it would need to be cleaned too.

Measurement with drops ascending

- Select the polarity of the voltage U, e.g. top plate "+", bottom plate"-".
- If times t_1 and t_2 have been saved from previously, they can be reset to zero by pressing "Reset".
- Produce, observe and select a suitable droplet as described above.
- Set voltage switch U to ON. Set up a voltage U at which the oil drop slowly rises past a selected initial scale marking in the top part of the measurement cell.
- Move switch U to OFF so that the oil drop starts to descend again.
- Set switch t to ON as soon as the oil drop has returned to the first position you chose. This starts the measurement of time t_2.
- Set switch U to ON as soon as the drop passes a selected scale marking in the lower part of the measurement cell, thus causing the drop to start rising again. Measurement of time t_2 stops and measurement of time t_1 starts automatically.
- Set switch t to OFF as soon as the oil drop has returned to the first position you chose. This causes the measurement of time t_1 to stop.
- Set switch U back to OFF.
- Read off the times t_1 and t_2 as well as the voltage U ("Previous Voltage") from the display and make a note of them along with the separation of the scale markings.
- Repeat the measurement for as many different oil drops as possible.

Tab. 1:Charges q_i determined from measurements of 10 different oil drops and the resulting values for elementary charge e_i.

i	t_{1i} s	t_{2i} s	U_i V	Polarität	r_i µm	q_i 10^{-19} C	Δq_i 10^{-19} C	n_i	e_i 10^{-19} C	Δe_i 10^{-19} C
1	12.426	13.780	107.0	+/−	0.81	-11.1	0.9	-7	1.59	0.13
2	14.414	17.433	109.4	+/−	0.71	-7.9	0.6	-5	1.58	0.12
3	13.604	9.053	292.6	+/−	1.00	-6.2	0.4	-4	1.55	0.10
4	13.641	23.631	190.9	−/+	0.61	3.5	0.2	2	1.75	0.10
5	10.502	14.858	246.1	−/+	0.78	4.9	0.3	3	1.63	0.10
6	14.203	21.674	110.9	−/+	0.64	6.3	0.5	4	1.58	0.13
7	9.814	10.228	279.4	−/+	0.94	6.6	0.4	4	1.65	0.10
8	13.813	16.824	120.4	−/+	0.73	7.6	0.6	5	1.52	0.12
9	9.936	16.380	112.1	−/+	0.74	10.2	0.8	6	1.70	0.13
10	13.184	12.214	124.5	−/+	0.86	10.6	0.8	7	1.51	0.11

SAMPLE MEASUREMENT AND EVALUATION

The following parameters are relevant to all subsequent evaluations

Distance d between capacitor plates:	3 mm
Distance moved by drop s (between top mark 6 and bottom mark 4 on ocular scale):	1 mm
Viscosity of air η	$1.876 \cdot 10^{-5}$ kg/(m·s)
Density of air ρ_1 (25°C, 1013 hPa)	1.184 kg/m^3
Density of oil ρ_2 (25°C)	871 kg/m^3
Acceleration due to gravity g	9.81 m/s
Air pressure p	1014 hPa
Correction parameter b	82 µm·hPa
Correction parameter A	8.1 10^{-8} m

Note:

Viscosity and air pressure should remain constant during all the measurements. If there is no guarantee that this is the case, e.g. the experiment is carried out with several measurements on different days, the corresponding values will need to be taken into account for each individual measurement.

Measurement errors

Distance between capacitor plates, Δd:	0.1 mm
Distance moved by drop, Δs	50 µm
Time (quartz controlled), Δt	1 µs
Voltage, ΔU (0.5% of maximum value 1000 V ±5 digits)	5.5 V

The measurement errors for the material, environmental and correction parameters are not significant and may therefore be ignored.

The uncertainties with the most significance are those for the distance between capacitor plates, Δd, and the distance between the selected marks on the ocular scale, Δs.

Determination of velocity and charge

- From the times t_1 and t_2 it takes the drops to ascend and descend it is possible to determine the drops' velocities:

  (11) $v_{1,2} = \frac{s}{V \cdot t_{1,2}}$,

  s: Distance travelled by drops between two selected marks on the ocular scale, $V = 2$: magnification of objective lens

Then equation (10) can be used to determine the charge q on the drops (Table 1).

Tab. 2: Determination of smallest integer arising from the product of the charge ratio q_j/q_k = 1.4 and the given integers n_k..

n_k	$1{,}4 \cdot n_k$
1	1,4
2	2,8
3	4,2
4	5,6
5	**7,0**
6	8,4
7	9,8
8	11,2
9	12,6
10	14,0

Determination of *n*

If there is such a thing as an elementary charge e, for all the measured charges q_j and q_k (j, k = 1, 2, 3, …, 10) the following must be true for a pair of oil drops:

$$(12)\quad q_j = n_j \cdot e \text{ and } q_k = n_k \cdot e \text{ where } n_j, n_k \in \mathbb{Z}$$

Therefore:

$$(13)\quad \frac{q_j}{q_k} = \frac{n_j}{n_k} \Leftrightarrow n_j = \frac{q_j}{q_k} \cdot n_k$$

The integers n_j and n_k can be determined as follows, whereby it is assumed without constraint that $|q_j| > |q_k|$:

- From the measured charges (Tab. 1) form ratios from pairs of results q_j/ q_k where $|q_j| > |q_k|$.
- Select a set of different charged pairs with roughly similar charge ratios (within the accuracy of measurements in the experiment).

The pairs of charges (q_1,q_2), (q_4,q_5) and (q_8,q_{10}) with charge ratios≈ 1.4, for example, make up such a set and will be used as an example in the following treatment.

- Try out the integers n_k = 1, 2, 3, … and successively calculate $1.4 \cdot n_k$ (Table 2). From these calculations, determine the value corresponding to the *smallest* integer or closest to the *smallest* integer. This value is then assigned to n_j.

The smallest integer from Table 2 is n_j = 7 when n_k = 5. Since the magnitudes of the charges q_1 and q_{10} or q_2 and q_8 are the same to within the limits of accuracy and those of q_4 and q_5 are smaller by a factor ≈ 2.3, the charge pair (q_1,q_2) is assigned the pair of values (n_1,n_2) = (−7,−5) and the charge pair (q_8,q_{10}) is given the pair of values (n_8,n_{10}) = (5,7).

Charge pair (q_4,q_5) is assigned the pair of values (n_4,n_5) = (2,3) because 7/2.3 = 3.0 and 5/2.3 = 2.2.

- Determine the values of n_i for the other charges from the ratios to a charge with the previously determined integer value n_i using equation (13). Enter all values for n_i into Table 1.

Example for determining n_6 from n_1: q_1/q_6 = −1.8 and n_1/−1.8 = 3.9, meaning that n_6 is assigned the value 4.

Alternatively the charges q_i determined from the measurements (Table 1) can each be divided by the integers n_i in such a way that the resulting values exhibit the minimum spread around the (non-weighted) mean. The standard deviation is the measure of this spread.

Determination of *e*

- Divide each of the charges q_i and the associated measurement errors Δq_i by n_i and thus obtain values of e_i and Δe_i for the elementary charge and its measurement error for each individual set of measurements (Table 1)
- Obtain the best estimate for elementary charge e and its standard deviation Δe from the values e_i gained in the individual measurements and their measurement errors Δe_i by forming a weighted mean:

$$(14)\quad e \pm \Delta e = \frac{\sum w_i \cdot e_i}{\sum w_i} \pm \frac{1}{\sqrt{\sum w_i}} \text{ where } w_i = \left(\frac{1}{\Delta e_i}\right)^2 .$$

Using the values in Table 1, the following is obtained:

$$(15)\quad \begin{aligned} e \pm \Delta e &= \left(\frac{1286}{799} \pm \frac{1}{28}\right) \cdot 10^{-19}\ \text{C} \\ &= (1.61 \pm 0.04) \cdot 10^{-19}\ \text{C} \end{aligned} .$$

The result is therefore all the more significant, the greater the number of measurements that are made, i.e. the larger the quantity of samples and the smaller the number n of differing charges on the drops. Due to measurement uncertainties, particularly in the distance between capacitor plates and readings from the microscope scale, it would be expected that $n \le 7$.