PHYSICS EXPERIMENT

Lens Equation

Determine the focal length of a lens using the Bessel method

- Determine the two positions of a thin lens where a sharp image is formed.
- Determine the focal length of a thin lens.

UE4010100

07/16 JöS

Fig. 1 Measurement set-up

GENERAL PRINCIPLES

The focal length *f* of a lens refers to the distance between the main plane of the lens and its focal point, see Fig. 2. This can be determined using the Bessel method (devised by *Friedrich Wilhelm Bessel*). This involves measuring the various separations between the optical components on the optical bench.

From Fig. 2 and Fig. 3 it can be seen that the following relationship must apply for a thin lens:

(1) $a = b + g$

a: distance between object *G* and image *B*
b: Distance between lens and image *B*
g: Distance between object *G* and lens

By plugging $b = a - g$ into the lens equation

(2) $$\frac{1}{f} = \frac{1}{b} + \frac{1}{g}$$

f: Focal length of lens

the following is obtained:

(3) $$\frac{1}{f} = \frac{a}{a \cdot g - g^2}$$

This corresponds to a quadratic equation $g^2 - a \cdot g + a \cdot f = 0$ with the following pair of solutions:

(4) $$g_{1,2} = \frac{a}{2} \pm \sqrt{\frac{a^2}{4} - a \cdot f}\ .$$

For both object distances g_1 and g_2, a well focussed image is obtained when $a > 4f$. The difference e between them allows the focal length to be determined:

$$(5) \quad e = g_1 - g_2 = \sqrt{a^2 - 4af}$$

The difference e is the difference between the two lens positions P1 and P2, which result in a focussed image.

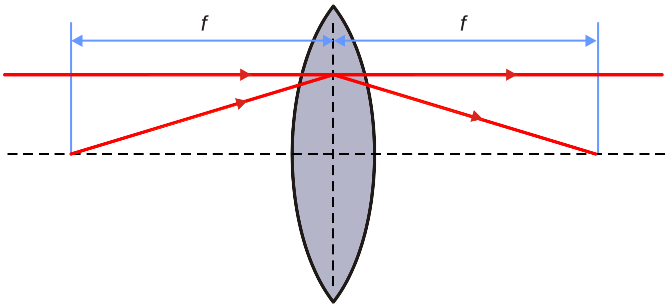

Fig. 2 Schematic showing the definition of focal length for a thin lens

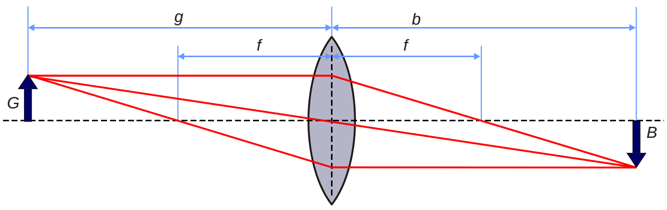

Fig. 3 Schematic of ray paths through a lens

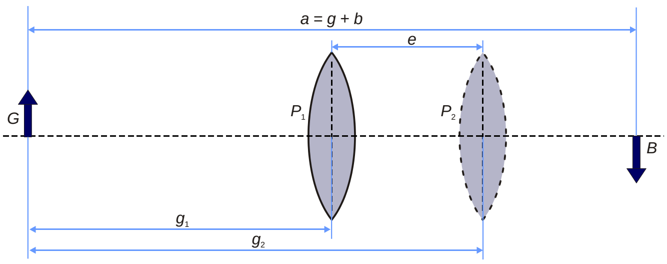

Fig. 4 Schematic showing the two lens positions which result in a well focussed image on the screen

LIST OF EQUIPMENT

1	Optical Bench K, 1000 mm	1009696 (U8475240)
4	Optical Rider K	1000862 (U8475350)
1	Optical Lamp K	1000863 (U8475400)
1	Transformer 12 V, 25 VA @230V	1000866 (U8475470-230)
or		
1	Transformer 12 V, 25 VA @115V	1000865 (U8475470-115)
1	Convex Lens K, f = 50 mm	1000869 (U8475901)
1	Convex Lens K, f = 100 mm	1010300 (U8475911)
1	Clamp K	1008518 (U84755401)
1	Set of 4 Image Objects	1000886 (U8476605)
1	Projection Screen K, White	1000879 (U8476320)

SET UP AND PROCEDURE

- Set up the four optical riders on the optical bench at positions -5 cm, 4 cm, 50 cm and 89.5 cm (with reference to the left-hand edge of the bench) and secure them in place. Insert the optical lamp into the first rider, one converging lens (f = 50 mm) along with the lens clamp into the second and the screen into the fourth and last, as shown in Fig. 1. Nothing should be inserted into the third rider yet.
- Connect the optical lamp to the 12 V transformer and turn it on.
- Move the second optical rider in such a way that a sharp image of the lamp filament can be seen on the screen.
- Insert the F-shaped aperture or the picture slide from the set of four image objects into the image clamp. Make sure that the illumination is uniform.
- Put the other converging lens (f = 100 mm) into the third optical rider.
- Move the converging lens (f = 100 mm) in short steps to find the two positions where a well focussed image appears on the screen.
- Read off the distance *a* between object and image, i.e. the difference between the positions of the object and the position of the screen on the optical bench's ruler, and enter the value into Table 1.
- Read off the two object distances g_1 and g_2 as the difference between the two positions of the converging lens (f = 100 mm) and the position of the object on the optical bench's ruler and enter the value into Table 1.
- By varying the position of the screen, repeat the measurement for different values of *a*. Make sure you keep to the condition $a > 4f$ (f = 100 mm) and start by adjusting the position of the second optical rider with the first converging lens (f = 50 mm) in it until another sharp image of the filament in the optical lamp can be seen on the screen.

SAMPLE MEASUREMENT AND EVALUATION

Tab. 1: Measured object distances g_1 und g_2, the difference between them *e* and calculations of focal length *f* for various distances *a* between the screen and the object

a / mm	g_1 / mm	g_2 / mm	*e* / mm	*f* / mm
826	714	118	596	99
724	605	124	481	101
674	556	130	426	101
613	487	138	349	104
522	394	134	260	98

A formula for the focal length of a thin lens can be derived using the Bessel method from equation

$$(6)\quad f = \frac{a^2 - e^2}{4a}$$

- Calculate the focal lengths *f* from the object-screen distances *a* and differences *e* between the object distances (Table 1) using equation (6) and enter the results into Table 1.
- Calculate the average (mean) of these results for focal length:

$$(7)\quad \bar{f} = \frac{\sum_{i=1}^{5} f_i}{5}$$

The average is *f* = 101 mm, which agrees very well with the nominal value of the lens (*f* = 100 mm).

Assuming that there will be an error of 1 mm in the positioning of the optical components and in the readings for their position on the scale rule of the optical bench, the relative uncertainty in individual measurements is about 1%.