

AC Resistance

DETERMINING IMPEDANCE IN A CIRCUIT WITH AN INDUCTIVE AND A CAPACITIVE REACTANCE.

- Determining the impedances of series and parallel connections of capacitive and inductive reactances as a function of frequency.
- Determining resonant frequency as a function of inductance and capacitance.
- Observing changes in phase shift between voltage and current at the resonant frequency.

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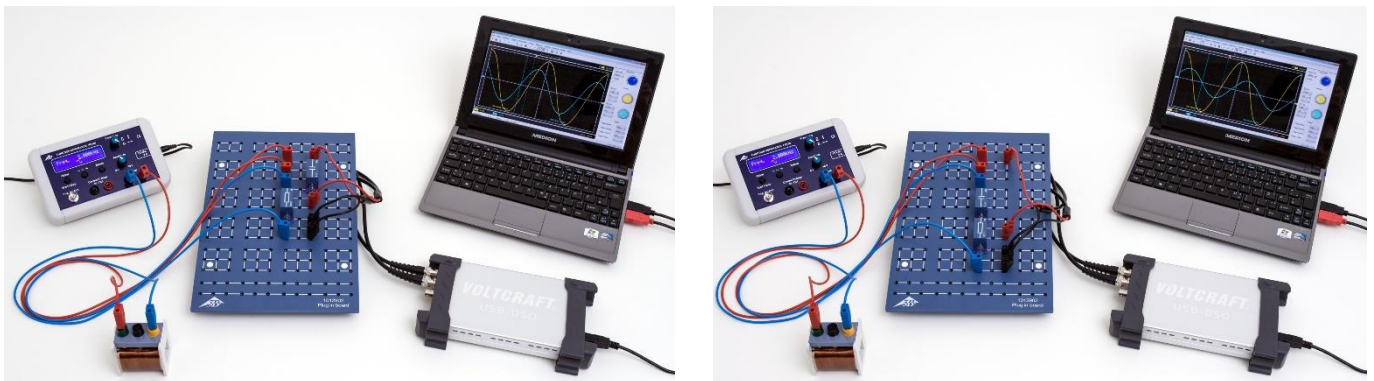


Fig. 1: Measurement set-up for series circuit (left) and parallel circuit (right).

BASIC PRINCIPLES

As the frequency of an AC circuit's current rises, the inductive reactance rises too, while the capacitive reactance drops. Series and parallel connections of capacitive and inductive reactances therefore exhibit resonant behaviour. One speaks here of a resonant circuit, its current and voltage oscillating back and forth between the capacitance and inductance. An additional ohmic resistor dampens these oscillations.

To simplify calculations for series and parallel connections, inductances L are assigned the following complex reactance:

$$(1) \quad X_L = i \cdot 2\pi \cdot f \cdot L.$$

f : Alternating current's frequency

Furthermore, capacitances C are assigned the following complex reactance:

$$(2) \quad X_C = \frac{1}{i \cdot 2\pi \cdot f \cdot C}.$$

The total impedance of a series connection without an ohmic resistance therefore is:

$$(3) \quad Z_S = i \cdot \left(2\pi \cdot f \cdot L - \frac{1}{2\pi \cdot f \cdot C} \right) = i \cdot Z_{S0}.$$

The corresponding calculation for a parallel connection is:

$$(4) \quad \frac{1}{Z_P} = -i \cdot \left(\frac{1}{2 \cdot \pi \cdot f \cdot L} - 2 \cdot \pi \cdot f \cdot C \right) = i \cdot \left(-\frac{1}{Z_{P0}} \right).$$

At the resonant frequency

$$(5) \quad f_r = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}}$$

the impedance Z_S of the series connection comprising inductive and capacitive reactances therefore vanishes, i.e. the voltages across both individual reactances are opposite and equal.

By contrast, the value of a parallel connection's impedance Z_P becomes infinite, i.e. the individual currents are opposite and equal. At the resonant frequency, the sign of the phase shift between the voltage and current furthermore changes.

In the experiment, resonant circuits are set up as series / parallel connections of capacitors and inductors. A function generator serves as a voltage source with an adjustable frequency and amplitude. An oscilloscope is used to measure current and voltage as functions of the set frequency. The voltage U and current I are displayed on the oscilloscope; I corresponds to the voltage drop $U_m(t)$ across a load resistor R_m (Fig. 2, 3).

LIST OF EQUIPMENT

1	Plug-In Board for Components	1012902 (U33250)
1	Capacitor 1 μ F, 100 V, P2W19	1012955 (U333063)
1	Capacitor 4.7 μ F, 63 V, P2W19	1012946 (U333054)
1	Coil S with 800 Taps	1001001 (U8498080)
1	Coil S with 1200 Taps	1001002 (U8498085)
1	Resistor 10 Ω , 2 W, P2W19	1012904 (U333012)
1	Function Generator FG 100 @230V	1009957 (U8533600-230)
or	@115V	1009956 (U8533600-115)
1	PC Oscilloscope 2x50 MHz	1020857 (U11830)
2	HF Patch Cord, BNC/4 mm Plug	1002748 (U11257)
1	Set of 15 Experiment Leads, 75 cm 1 mm ²	1002840 (U13800)

SET UP AND PROCEDURE

Series circuit

- Assemble a set-up for measurement of the series circuit (as in Fig. 1) in accordance with the sketch of the circuit diagram (Fig. 2) in which $R_m = 10 \Omega$, $C = 1 \mu F$ and the coil has $N = 1200$ turns and inductance $L = 23.0$ mH.
- Connect the output signal $U_m(t) = I(t) \cdot R_m$ to channel 1 (CH1) of the oscilloscope and the input signal $U(t)$ to channel 2 (CH2).
- Configure the following starting parameters for the PC oscilloscope:

Time/DIV	10 μ s
Volts/DIV CH1	10.0 mV AC
Volts/DIV CH2	2.00 V AC
Trigger Mode	Auto
Sweep	Edge
Source	CH2
Slope	+
Level	0.00 μ V

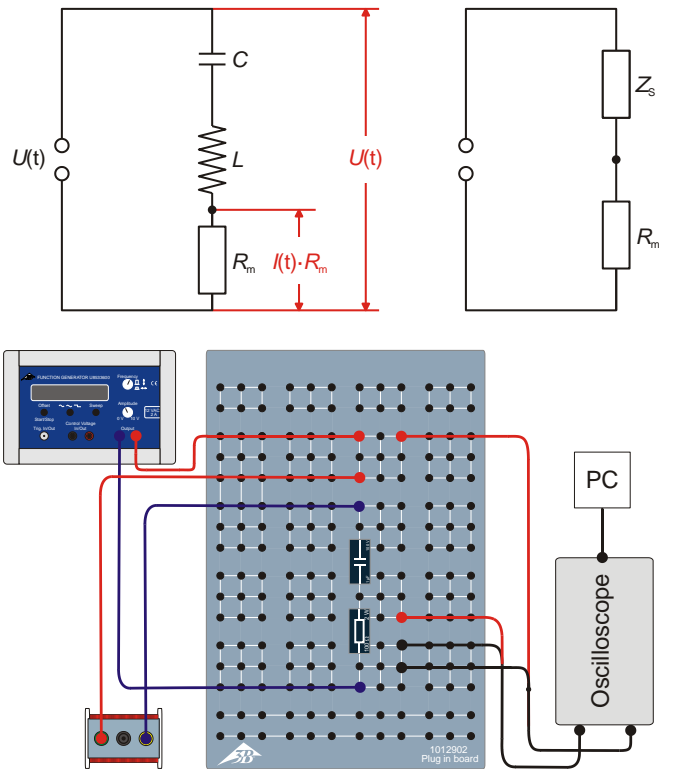


Fig. 2: Circuit diagram (top left), equivalent circuit (top right) and schematic sketch of set-up (below) for series circuit.

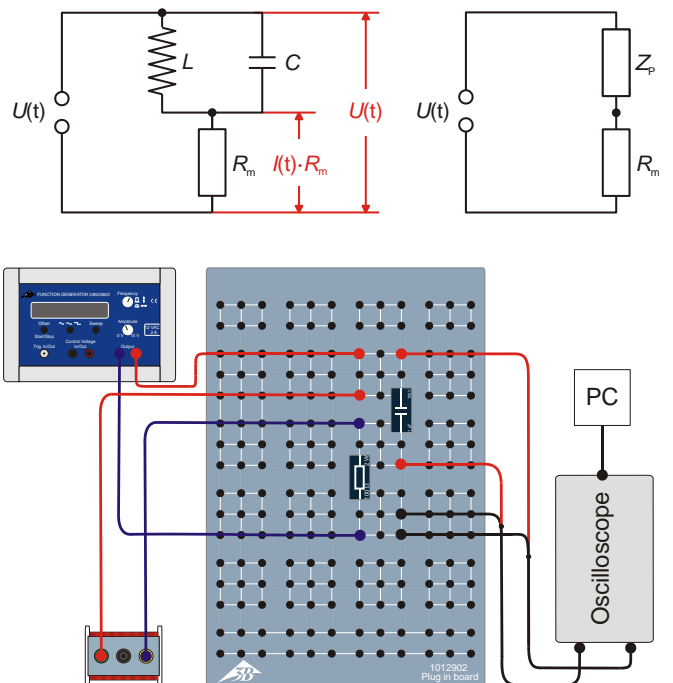


Fig. 3: Circuit diagram (top left), equivalent circuit (top right) and schematic sketch of set-up (below) for parallel circuit.

Note

The parameters Time/DIV and Volts/DIV for CH1 will need to be modified during the course of this measurement sequence.

- Select Sine as the signal form for the function generator and set the amplitude of the input signal to $U_0 = 6\text{ V}$. Adjust the amplitude knob in such a way that the maximum or minimum of the sine-wave signal on channel CH2 of the scope (with 2 V per division) coincides with exactly 3 divisions.
- Set a frequency of 20000 Hz on the function generator and reduce it step by step down to 50 Hz (Table 1). For each frequency, read off the amplitude U_{m0} of the output signal $U_m(t)$ from the oscilloscope and enter these values into Table 1.
- Determine the resonant frequency f_r , the frequency at which U_{m0} is highest. Enter the values of f_r and U_{m0} into Table 1 (marked in bold right).
- Repeat the measurements for a coil with $N = 800$ turns and inductance $L = 10.4\text{ mH}$.
- Repeat the measurements for both coils with a capacitor where $C = 4.7\text{ }\mu\text{F}$.
- Enter all the resonant frequencies into Table 2.

SAMPLE MEASUREMENT

Tab. 1: Set frequencies f and amplitude measurements of U_{m0} for output signal $U_m(t)$ in series and parallel circuits, $R_m = 10\text{ }\Omega$, $C = 1\text{ }\mu\text{F}$, $N = 1200$ turns / $L = 23.0\text{ mH}$.

f / Hz	U_{m0} / V	
	Series circuit	Parallel circuit
20000	0.0194	4.5060
10000	0.0397	3.1030
5000	0.0826	1.6900
2000	0.2800	0.5350
1500	0.5080	0.2890
1052	1.9950	0.0487
1000	1.8470	0.0597
800	0.6800	0.2120
500	0.2400	0.5880
200	0.0773	1.4080
50	0.0193	1.9950

Parallel circuit

- Assemble a set-up for measurement of the parallel circuit (as in Fig. 1) in accordance with the sketch of the circuit diagram (Fig. 3) in which $R_m = 10\text{ }\Omega$, $C = 1\text{ }\mu\text{F}$ and the coil has $N = 1200$ turns and inductance $L = 23.0\text{ mH}$.
- Carry out the measurements in the same way as for the series circuit. Use the same initial parameters for the USB oscilloscope but set Volts/DIV for CH1 to 2.00 V AC.
- Determine the resonant frequency f_r , the frequency at which U_{m0} is lowest. Enter the values of f_r and U_{m0} into Table 1 (marked in bold).
- Enter all the resonant frequencies into Table 3.

Tab. 2: Resonant frequencies measured for series circuit.

	f_r	
	$C = 1\text{ }\mu\text{F}$	$C = 4.7\text{ }\mu\text{F}$
$N = 1200$ $L = 23.0\text{ mH}$	1052 Hz	493 Hz
$N = 800$ $L = 10.4\text{ mH}$	1471 Hz	690 Hz

Phase shift

- Assemble the set-up for measurement of the series circuit (as in Fig. 1) in accordance with the sketch of the circuit diagram (Fig. 2) in which $R_m = 10\text{ }\Omega$, $C = 1\text{ }\mu\text{F}$ and the coil has $N = 1200$ turns and inductance $L = 23.0\text{ mH}$.
- Set up the resonant frequency f_r , then increase and decrease the frequency a little and observe what happens to the phase.
- Repeat the measurements for the coil with $N = 800$ turns and inductance $L = 10.4\text{ mH}$.
- Repeat the measurements for both coils with the capacitor $C = 4.7\text{ }\mu\text{F}$.
- Repeat the measurements for the parallel circuit.

Tab. 3: Resonant frequencies measured for parallel circuit.

	f_r	
	$C = 1\text{ }\mu\text{F}$	$C = 4.7\text{ }\mu\text{F}$
$N = 1200$ $L = 23.0\text{ mH}$	1052 Hz	496 Hz
$N = 800$ $L = 10.4\text{ mH}$	1457 Hz	688 Hz

EVALUATION

- Calculate the sums Z_0 for the total impedance of both the series and parallel circuit from the measured voltages U_{m0} (Table 1).

$$(9) \quad I_0 = \frac{U_{m0}}{R_m} = \frac{(U_0 - U_{m0})}{Z_0} \Leftrightarrow Z_0 = \left(\frac{U_0}{U_{m0}} - 1 \right) \cdot R_m,$$

$U_0 = 6 \text{ V}$, $R_m = 10 \text{ } \Omega$. Enter the values into Table 4.

- Plot the sums Z_{0S} and Z_{0P} for the total impedance of both the series and parallel circuit as function of frequency f (Fig. 4, Fig. 5).
- Calculate the theoretical values of the sums Z_{0S} and Z_{0P} for the total impedance of both the series and parallel circuit with the help of equations (3) and (4) and draw the results as continuous lines in both Fig. 4 and Fig. 5.
- Calculate the resonant frequencies using equation (5) and enter them into Table 5. Then plot the measured resonant frequencies (tables 2, 3) along with the calculated ones (Fig. 6).

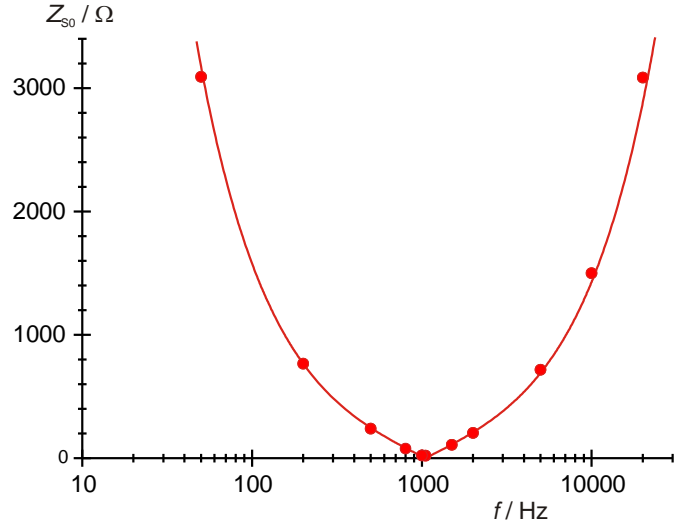


Fig. 4: Impedance of a series connection as a function of frequency

Tab. 4: Total impedance values Z_{0S} and Z_{0P} obtained from measurement of serial and parallel circuits.

f / Hz	Z_{0S} / Ω	Z_{0P} / Ω
20000	3082.8	3.3
10000	1501.3	9.3
5000	716.4	25.5
2000	204.3	102.1
1500	108.1	197.6
1052	20.1	1222.0
1000	22.5	995.0
800	78.2	273.0
500	240.0	92.0
200	766.2	32.6
50	3098.8	20.1

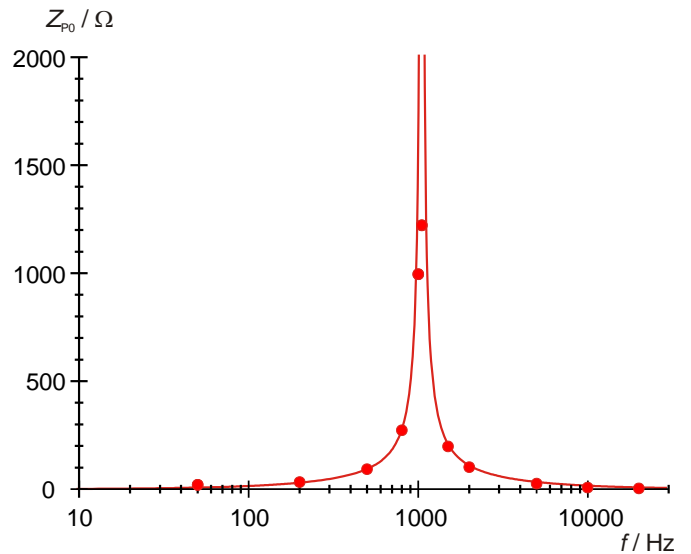


Fig. 5: Impedance of a parallel connection as a function of frequency

Tab. 5: Resonant frequencies calculated from capacitance and inductance.

	$f_r = 1 / (2 \cdot \pi \cdot \sqrt{L \cdot C})$	
	$C = 1 \text{ } \mu\text{F}$	$C = 4.7 \text{ } \mu\text{F}$
$N = 1200$ $L = 23.0 \text{ mH}$	1049 Hz	484 Hz
$N = 800$ $L = 10.4 \text{ mH}$	1576 Hz	727 Hz

The resonance curves as measured (Fig. 4, Fig. 5) provide confirmation of the theoretically expected characteristic (continuous lines). The large discrepancies in the sum total impedance values can be explained by the fact that coils also have an ohmic resistance component.

The measured resonant frequencies are well in agreement with the calculated ones (Fig. 6).

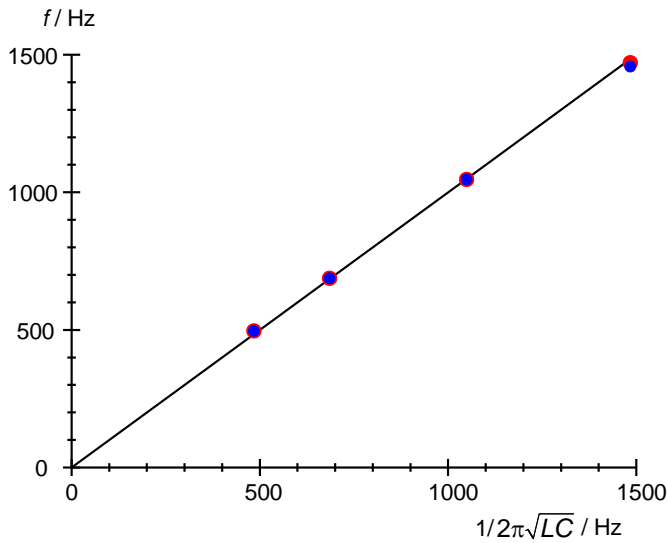


Fig. 6: Comparison between measured and calculated resonant frequencies for a series connection (red) and a parallel connection (blue).

Phase shift

For the series circuit, the maxima, minima and zero crossings in the output signal $U_m(t) = I(t) \cdot R_m$ are in the same places along the time axis as in the input signal $U(t)$, i.e. the phase shift $\varphi = 0^\circ$. If the frequency is increased or decreased, the output signal shifts leftward or rightward of the input signal as far as $|\varphi| = 90^\circ$.

For the series circuit, the maxima, minima and zero crossings in the output signal $U_m(t) = I(t) \cdot R_m$ are in the same places along the time axis as in the input signal $U(t)$, i.e. the phase shift $\varphi = 0^\circ$. If the frequency is increased or decreased, the output signal shifts rightward or leftward of the input signal as far as $|\varphi| = 90^\circ$, i.e. the phase shift is in the opposite direction to that for the series circuit. According to theory, the output signal would be expected to be zero at the resonant frequency because then the current would be zero. There would therefore be no observable phase and no phase shift at this point. The fact that the output signal differs from zero and there is indeed an observable phase and phase shift can be explained by the ohmic resistance component of the coils.

