AC Resistance

DETERMINE THE AC RESISTANCE IN A CIRCUIT WITH CAPACITIVE AND RESISTIVE LOADS.

- **Determine the amplitude and phase of the overall resistance as a function of frequency for a series circuit.**
- **Determine the amplitude and phase of the overall resistance as a function of frequency for a parallel circuit.**

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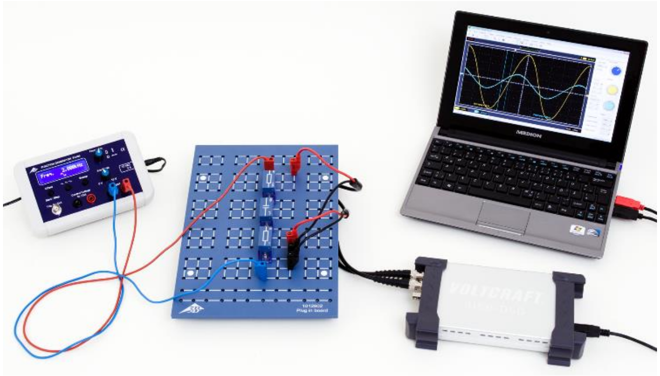

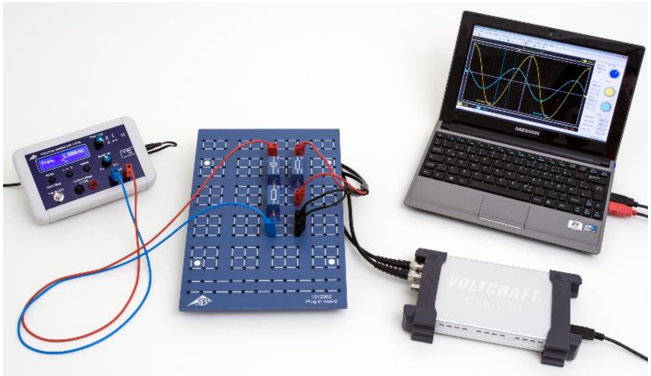

Fig. 1: Measurement set-up for series circuit (left) and parallel circuit (right).

GENERAL PRINCIPLES

In AC circuits, it is common to use complex numbers to describe the resistance in circuits with capacitors because this actually makes calculation easier. This is because not only the amplitude of the current and voltage is a factor, but also the phase relationships between the two need to be taken into account (this complex resistance is usually called impedance). Series and parallel circuits with both ohmic and capacitive resistance can then be described quite easily, although in each case, only the real component is measurable).

The complex resistance (impedance) of a capacitor with capacitance C in a circuit with an alternating current of frequency f is as follows:

$$(1)\quad X_C = -i \cdot X_{C0} = -i \cdot \left(\frac{1}{\omega \cdot C}\right) = \frac{1}{i \cdot \omega \cdot C},$$

Angular frequency $\omega = 2\pi \cdot f$

Therefore series circuits containing a capacitor and an ohmic resistor R will have the following overall resistance:

$$(2)\quad Z_S = \frac{1}{i \cdot \omega \cdot C} + R,$$

A parallel circuit can be assigned the following overall resistance

$$(3)\quad Z_P = \frac{1}{i \cdot \omega \cdot C + \frac{1}{R}}$$

The usual way of expressing this is as follows:

$$(4)\quad Z = Z_0 \cdot \exp(i \cdot \varphi)$$

This becomes

$$(5)\quad \begin{aligned} Z_S &= Z_{S0} \cdot \exp(i \cdot \varphi_S) \\ &= \frac{\sqrt{1 + (\omega \cdot C \cdot R)^2}}{\omega \cdot C} \cdot \exp(i \cdot \varphi_S) \end{aligned}$$

where $\tan\varphi_S = -\frac{1}{\omega \cdot C \cdot R}$

and

$$Z_P = Z_{P0} \cdot \exp(i \cdot \varphi_P) = \frac{R}{\sqrt{1+(\omega \cdot C \cdot R)^2}} \cdot \exp(i \cdot \varphi_P) \tag{6}$$

where $\tan\varphi_P = -\omega \cdot C \cdot R$.

Assume the total impedance $Z = Z_S$ or Z_P has the following voltage applied across it:

$$U = U_0 \cdot \exp(i \cdot 2 \cdot \pi \cdot f \cdot t) \tag{7}$$

The current which flows would then be as follows:

$$I = \frac{U_0}{Z_0} \cdot \exp(i \cdot (2 \cdot \pi \cdot f \cdot t - \varphi)) = I_0 \cdot \exp(i \cdot (2 \cdot \pi \cdot f \cdot t - \varphi)) \tag{8}$$

In the experiment this current is determined by measuring the drop in voltage $U_m(t)$ across a resistor R_m (Fig. 2, 3). The resistance value for this resistor is chosen such that $U_{m0} << U_0$, i.e. nearly all of the applied voltage drops across Z_S or Z_P. The current so determined will be flowing through both Z_S and Z_P, since the two resistors are connected in series with R_m (see equivalent circuit in Figs. 2 and 3). Since $U_m(t) = I(t) \cdot R_m$, the change in voltage over time $U_m(t)$ reflects the change in current $I(t)$.

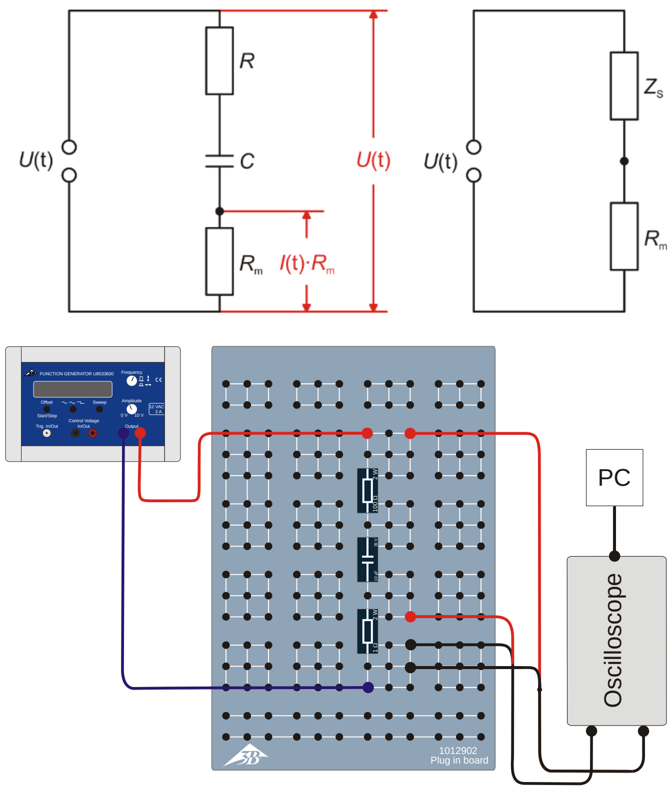

Fig. 2: Circuit diagram sketch (top left), equivalent circuit diagram (top right) and set-up schematic (bottom) for series circuit.

LIST OF EQUIPMENT

1 Plug-In Board for Components	1012902 (U33250)
1 Resistor 1 Ω, 2 W, P2W19	1012903 (U333011)
1 Resistor 100 Ω, 2 W, P2W19	1012910 (U333018)
1 Capacitor 10 µF, 35 V, P2W19	1012957 (U333065)
1 Capacitor 1 µF, 100 V, P2W19	1012955 (U333063)
1 Capacitor 0.1 µF, 100 V, P2W19	1012953 (U333061)
1 Function Generator FG 100 @230V	1009957 (U8533600-230)
or	
1 Function Generator FG 100 @115V	1009956 U8533600-115)
1 PC Oscilloscope, 2x25 MHz	1020857 (U11830)
2 HF Patch Cord, BNC/4 mm Plug	1002748 (U11257)
1 Set of 15 Experiment Leads, 1 mm²	1002840 (U13800)

SET-UP AND EXPERIMENT PROCEDURE

Series circuit

- Set up the equipment for measuring a series circuit (Fig. 1, left) as shown in the sketch of the circuit diagram (Fig. 2) with components $R_m = 1\ \Omega$, $R = 100\ \Omega$ and $C = 10\ \mu F$.
- Connect the output signal $U_m(t) = I(t) \cdot R_m$ to channel CH1 on the oscillscope and the input signal $U(t)$ to channel CH2.

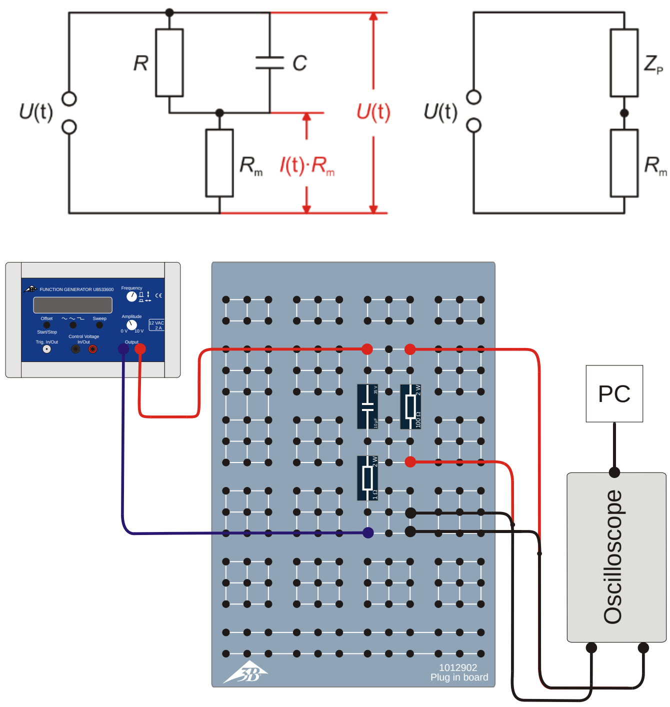

Fig. 3: Circuit diagram sketch (top left), equivalent circuit diagram (top right) and set-up schematic (bottom) for parallel circuit.

- Set up the PC oscilloscope with the following parameters:

Horizontal:	
Time base:	50 μs/div
Horizontal trigger position:	0.0 ns
Vertical:	
CH1:	
Voltage scale division:	20 mV/div DC
Zero position:	0.0 divs
CH2:	
Voltage scale division:	2 V/div DC
Zero position:	0.0 divs
Trigger:	
Single (not Alternate)	
Source:	CH2
Mode:	Edge
Edge:	Rising
Threshold:	0.000 mV
Trigger mode:	Auto

Note
It will be necessary to change the Time/div and Volts/div settings during the series of measurements.

- Select a sinusoidal wave form on the function generator and adjust the amplitude of the input signal to U_0 = 6 V. Set the amplitude control in such a way that the maximum and minimum of the sinusoidal signal on channel CH2 of the oscilloscope are separated by three divisions (for a setting of 2 V/div).
- Set up the following frequencies 2000 Hz, 1000 Hz, 500 Hz, 200 Hz, 100 Hz and 50 Hz one by one on the function generator. Calculate the corresponding period durations using the formula $T = 1/f$ and enter them into Table 1 along with the frequencies.
- Read off the amplitude U_{m0} of the output signal $U_m(t)$ from the oscilloscope and enter the values into Table 1.
- Read off the oscilloscope the time difference Δt between the places where the signals $U(t)$ und $U_m(t)$ cross the zero axis and enter the values into Table 1.
- Repeat the measurements using the capacitor of value C = 1 μF at the same frequencies and using the capacitor of value C = 0.1 μF at 2000 Hz and 1000 Hz. Enter all the results into Table 1.

Parallel circuit

- Set up the equipment for measuring a parallel circuit (Fig. 1, right) as shown in the sketch of the circuit diagram (Fig. 3) with components R_m =1 Ω, R =100 Ω and C = 10 μF.
- Carry out the measurements in a similar way to how they were done on the series circuit. Select the same initial parameters for the PC oscilloscope except that Volts/div on channel CH1 should be set to 200 mV DC.
- Enter all the measurements into Table 2.

SAMPLE MEASUREMENT AND EVALUATION

Tab. 1: Stipulated, measured and calculated values for series circuit, U_0 = 6 V, R_m =1 Ω.

C / µF	f / Hz	T / ms	X_{C0} / Ω	U_{m0} / mV	Δt / ms	I_0 / mA	Z_{S0} / Ω	φ_S
10.0	2000	0.5	8.0	56.9	0.006	56.9	105.4	4.3°
10.0	1000	1.0	15.9	56.7	0.026	56.7	105.8	9.4°
10.0	500	2.0	31.8	53.5	0.095	53.5	112.1	17.1°
10.0	200	5.0	79.6	42.8	0.500	42.8	140.2	36.0°
10.0	100	10.0	159.2	30.2	1.479	30.2	198.7	53.2°
10.0	50	20.0	318.3	17.9	3.689	17.9	335.2	66.4°
1.0	2000	0.5	79.6	45.8	0.055	45.8	131.0	39.6°
1.0	1000	1.0	159.2	31.1	0.157	31.1	192.9	56.5°
1.0	500	2.0	318.3	18.2	0.400	18.2	329.7	72.0°
1.0	200	5.0	795.8	7.0	1.153	7.0	857.1	83.0°
1.0	100	10.0	1591.5	4.1	2.517	4.1	1463.4	90.6°
0.1	2000	0.5	795.8	7.6	0.114	7.6	789.5	82.1°
0.1	1000	1.0	1591.5	3.8	0.229	3.8	1578.9	82.4°

Tab. 2: Stipulated, measured and calculated values for parallel circuit, U_0 = 6 V, R_m =1 Ω.

C / µF	f / Hz	T / ms	X_{C0} / Ω	U_{m0} / mV	Δt / ms	I_0 / mA	Z_{P0} / Ω	φ_P
10.0	2000	0.5	8.0	679.7	0.078	679.7	8.8	56.2
10.0	1000	1.0	15.9	360.9	0.194	360.9	16.6	69.8
10.0	500	2.0	31.8	190.9	0.359	190.9	31.4	64.6
10.0	200	5.0	79.6	96.4	0.507	96.4	62.2	36.5
10.0	100	10.0	159.2	71.1	0.826	71.1	84.4	29.7
10.0	50	20.0	318.3	62.5	0.893	62.5	96.0	16.1
1.0	2000	0.5	79.6	93.1	0.069	93.1	64.4	49.7
1.0	1000	1.0	159.2	70.2	0.081	70.2	85.5	29.2
1.0	500	2.0	318.3	61.5	0.086	61.5	97.6	15.5
1.0	200	5.0	795.8	59.2	0.073	59.2	101.4	5.3
1.0	100	10.0	1591.5	58.6	0.069	58.6	102.4	2.5
0.1	2000	0.5	795.8	60.1	0.010	60.1	99.8	7.2
0.1	1000	1.0	1591.5	58.2	0.010	58.2	103.1	3.6

- Calculate the magnitude of the capacitive impedance using the formula $X_{C0} = 1/(2 \cdot \pi \cdot f \cdot C)$ (se equation 1) and enter the values into tables 1 and 2.
- Calculate the values of current amplitude from the measurements of U_{m0} (tables 1 and 2) and R_m (1 Ω) using the formula $I_0 = U_{m0}/R_m$ and enter the results into tables 1 and 2.
- Calculate the magnitudes of the total impedance Z_{S0} and Z_{P0} using the formula $Z_0 = U_0/I_0$ (U_0 = 6 V) and enter the results into Table 3.
- Calculate the phase shift from the values of the period T and time differential Δt (tables 1 and 2) using the formula $\varphi = 360° \cdot \Delta t / T$. Enter the results into tables 1 and 2.
- Calculate the magnitudes of the total impedance Z_{S0} and Z_{P0} along with the phase shifts φ_S and φ_P for both series and parallel circuits. Plot them on a graph as a function of X_{C0} (Figs. 4 – 7). Calculate the theoretical magnitudes of the total impedance Z_{S0} and Z_{P0} along with the phase shifts φ_S and φ_P according to equation (5) for the series circuit and equation (6) for the parallel circuit.

$$(9) \quad Z_{S0} = \sqrt{R^2 + X_{C0}^{\ 2}}\ , \ \varphi_S = \arctan\left(-\frac{X_{C0}}{R}\right)$$

$$(10) \quad Z_{P0} = \frac{1}{\sqrt{\frac{1}{R^2} + \frac{1}{X_{C0}^{\ 2}}}}\ , \ \varphi_P = \arctan\left(-\frac{R}{X_{C0}}\right),$$

Draw lines fitted to the points in Figs. 4 – 7.

Summary

For small frequencies, the series circuit will exhibit a resistance equal to the capacitive impedance, while in the parallel circuit it will have the value of the ohmic resistance. Phase shift is between 0° and -90° and will be -45° when the capacitive impedance and ohmic resistance are equal.

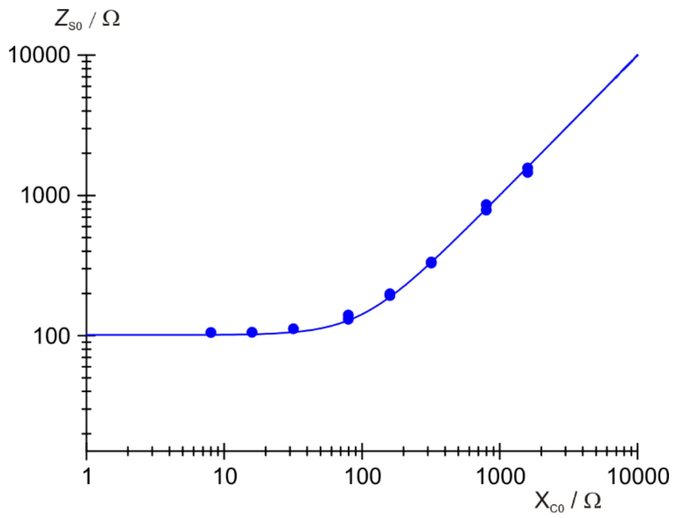

Fig. 4: Total impedance in series circuit.

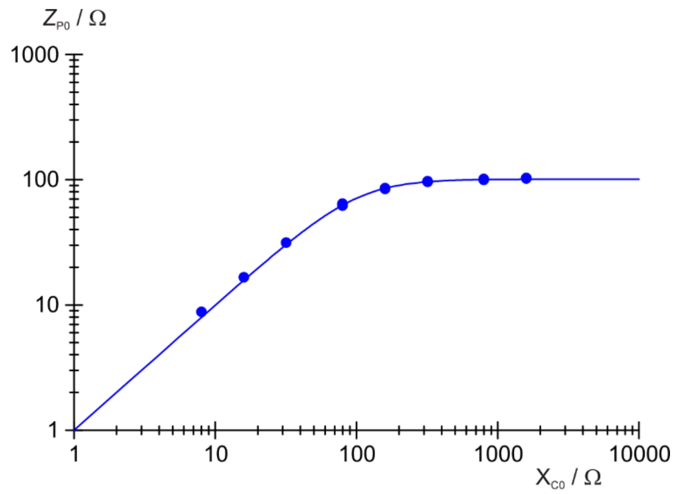

Fig. 6: Total impedance in parallel circuit.

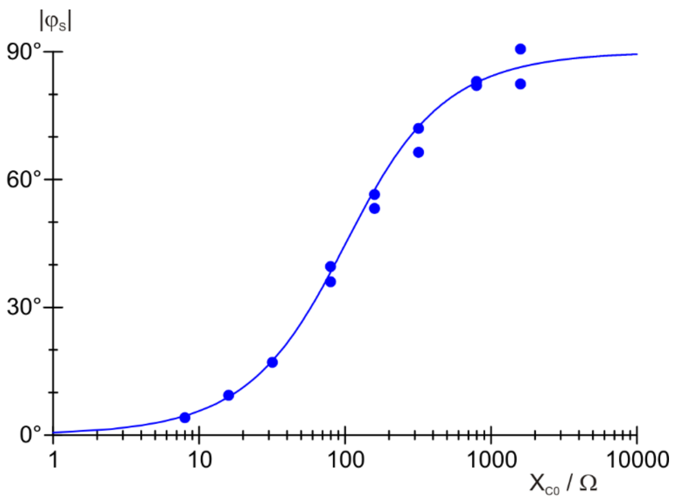

Fig. 5: Phase shift in series circuit.

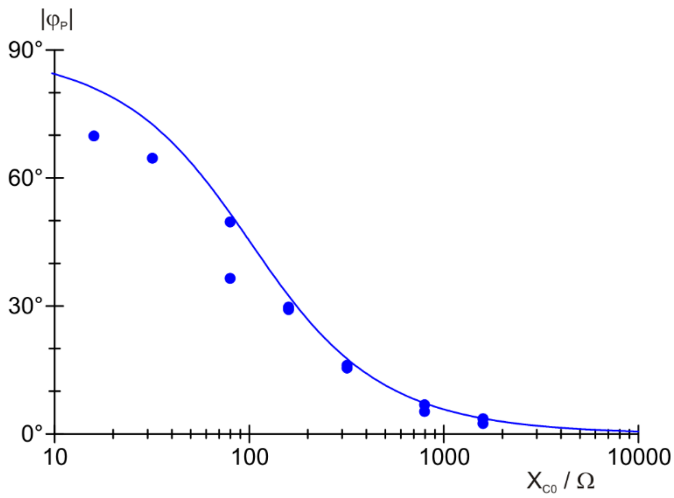

Fig. 7: Phase shift in parallel circuit.