

## Wheatstone's Bridge

### DETERMINE THE VALUE OF CERTAIN RESISTANCES

- Determine resistances using a Wheatstone bridge.
- Estimate the accuracy of the measurements.

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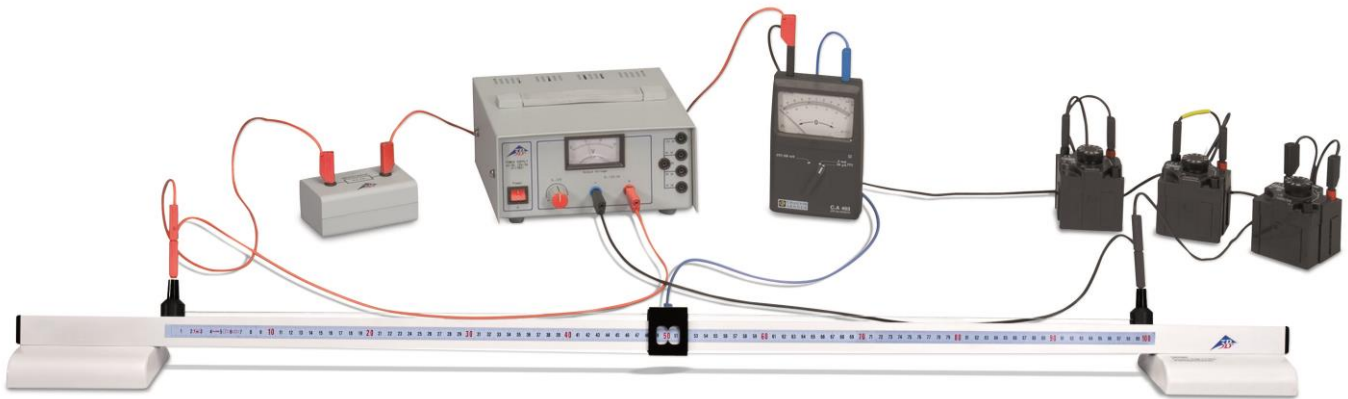


Fig. 1: Experiment set-up.

### BASIC PRINCIPLES

A classical method for measuring resistances uses a voltage balancing bridge named after *Charles Wheatstone* to compare the unknown resistance with a reference resistance. This involves setting up a circuit consisting of two voltage dividers in parallel, with a single DC voltage source connected across the whole. The first voltage divider consists of the resistance  $R_x$  that is to be measured and a reference resistance  $R_{ref}$ , while the second consists of two resistances  $R_1$  and  $R_2$ , the sum of which remains constant during the balancing process (see Fig. 2).

The ratio between the resistances  $R_1$  and  $R_2$  and – if necessary – the value of the reference resistance  $R_{ref}$  are varied until the current across the diagonal is reduced to zero. This occurs when the ratio between the resistances is the same for both voltage dividers. This balance condition leads to the following expression for the unknown resistance  $R_x$ :

$$(1) \quad R_x = R_{ref} \cdot \frac{R_1}{R_2}.$$

In this experiment the second voltage divider consists of a resistance wire 1 metre in length, which is divided into two sections of lengths  $s_1$  and  $s_2$  by a sliding contact.

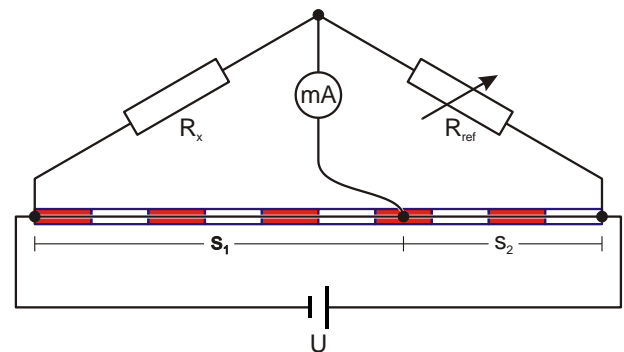


Fig. 2: Schematic diagram of a Wheatstone bridge

The two resistance values  $R_1$  und  $R_2$  are given by the following equation:

$$(2) \quad R_{1,2} = \rho \cdot \frac{s_{1,2}}{A}$$

$\rho$ : Resistivity of wire material  
 $A$ : Cross sectional area of resistance wire

This is because those two values are represented by the two parts of the resistance wire. Equation (1) can therefore be transformed as follows:

$$(3) \quad R_x = R_{ref} \cdot \frac{s_1}{s_2} = R_{ref} \cdot \frac{s_1}{(1m - s_1)},$$

The accuracy of the result depends on the tolerance of the reference resistance  $R_{ref}$ , the accuracy with which it was possible for the ratio of the two lengths of resistance  $s_1/s_2$  to be set up in order to obtain the resistance ratio  $R_1/R_2$  and how accurately it was possible to carry out the balancing of the bridge.

According to Gauss's theory on the propagation of uncertainty, the absolute error in the measurement is as follows:

$$(4) \quad \Delta R_x = \sqrt{\left(\frac{s_1}{(1m - s_1)} \cdot \Delta R_{ref}\right)^2 + \left(R_{ref} \cdot \frac{1m \cdot \Delta s_1}{(1m - s_1)^2}\right)^2}$$

This results in the following relative uncertainty:

$$(5) \quad \frac{\Delta R_x}{R_x} = \sqrt{\left(\frac{\Delta R_{ref}}{R_{ref}}\right)^2 + \left(\frac{\Delta s_1}{s_1} \cdot \frac{1m}{(1m - s_1)}\right)^2}$$

The relative uncertainty in the measurement  $\Delta R_x/R_x$  is shown in Figure 3 for the range  $0\text{ m} < s_1 < 1\text{ m}$ ,  $\Delta R_{ref}/R_{ref} = 0.005$  (0.5%) and a reading error of  $\Delta s_1 = \pm 0.5\text{ mm}$ . It is symmetrical with reference to  $s_1 = 0.5\text{ m}$ , where it is at a minimum and tends to infinity for  $s_1 \rightarrow 0\text{ m}$  and  $s_1 \rightarrow 1\text{ m}$ .

The accuracy of a Wheatstone bridge in this slider and wire configuration is therefore greatest when the sliding contact is positioned in the centre at  $s_1 = s_2 = 0.5\text{ m}$ . According to equation (3) it will then also be the case that  $R_x = R_{ref}$ . The reference resistance should therefore be sought out in such a way that the two sections of the wire  $s_1$  and  $s_2$  are the same length, i.e.  $s_1/s_2 = 1$ .

The accuracy with which the balancing of the bridge can be determined is described by the uncertainty in the balancing, which is inversely proportional to the sensitivity of the bridge, i.e. the more sensitive the bridge is, the more accurate the balance will be.

The sensitivity is given by the ratio between the full scale of the zero point galvanometer measuring the balance and the change in position of the sliding contact which causes the needle to deflect by the full scale. This increases with the sensitivity of the galvanometer and the supply voltage  $U$  for the measuring bridge. It also depends on the resistance values of the resistors in the bridge and the resistance of the zero point galvanometer. It is at a maximum when the sliding contact is at the centre of the resistance wire. This means that not only is the ratio  $s_1/s_2$  as good as it can be but so is the accuracy of the balancing.

Since the resistance of the wire used in the bridge is only about one order of magnitude greater than that of the conductors leading to it, resistors of value  $R_x \geq 100\ \Omega$  are used for the measurement.

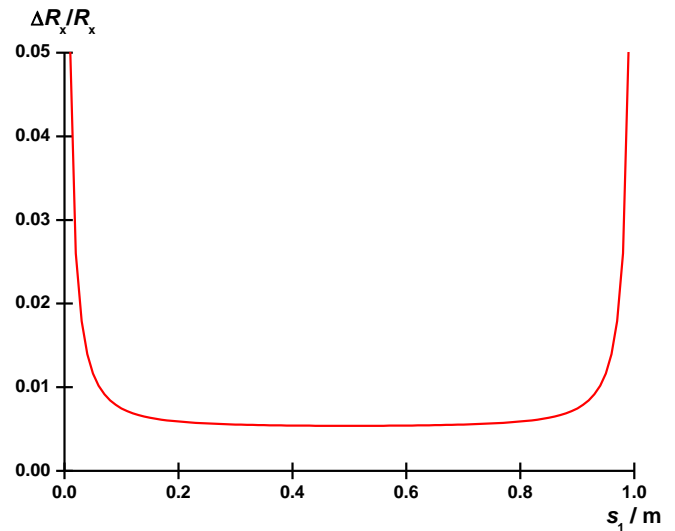


Fig. 3: Relative error  $\Delta R_x/R_x$  as a function of  $s_1$  according to equation (5) where  $\Delta R_{ref}/R_{ref} = 0.005$  (0.5%) and  $\Delta s_1 = \pm 0.5\text{ mm}$

### LIST OF EQUIPMENT

|  |                       |
|--|-----------------------|
| 1 Resistance Bridge                        | 1009885 (U8551002)    |
| 1 AC/DC Power Supply 0...12 V/ 3 A @230V   | 1002776 (U117601-230) |
| or   |                       |
| 1 AC/DC Power Supply 0...12 V/ 3 A @115V   | 1002775 (U117601-115) |
| 1 Zero Point Galvanometer CA 403           | 1002726 (U11170)      |
| 1 Resistance Decade 100 $\Omega$           | 1002732 (U11182)      |
| 1 Resistance Decade 1 k $\Omega$           | 1002733 (U11180)      |
| 1 Resistance Decade 10 k $\Omega$          | 1002734 (U11181)      |
| 1 Precision Resistor 100 $\Omega$          | 1009886 (U51004)      |
| 1 Precision Resistor 1 k $\Omega$          | 1009887 (U51005)      |
| 1 Set of 15 Safety Experiment Leads, 75 cm | 1002843 (U138021)     |

## SET UP AND PROCEDURE

### Safety instructions:

Do not exceed the maximum permissible voltage of 8 V or the maximum permissible current of 1.5 A.

Disconnect the power if the zero point galvanometer used for reading the balance is overloaded.

- Set up the experiment as shown in Fig. 1. Connect the black terminal on the right of the measuring bridge via one wire to the negative of the power supply and by another to the “COM” socket of the zero point galvanometer via the resistor decades connected in series. The red terminal on the left of the bridge should be connected via one wire to the positive terminal of the power supply and by another to the “COM” socket of the zero point galvanometer via precision resistors of value 100 Ω or 1 kΩ. The positive socket “+” of the zero point galvanometer should be connected to the sliding contact on the measuring bridge. Do not turn on the power supply yet.
- Select a 30 μA range on the zero point galvanometer and check that the needle is correctly pointing to zero. If necessary calibrate the zero point by turning the adjustment screw on the front panel.

The precision resistors act as the resistance  $R_x$  to be measured. The resistor decades are used for generating various fixed reference values  $R_{ref}$ .

The resistance  $R_x = 100 \Omega$  is measured with the help of reference resistors  $R_{ref} = 10 \Omega, 50 \Omega, 100 \Omega, 500 \Omega$  and 1 kΩ (Table 1), while resistance is measured  $R_x = 1 \text{ k}\Omega$  with  $R_{ref} = 100 \Omega, 500 \Omega, 1 \text{ k}\Omega, 5 \text{ k}\Omega$  and 10 kΩ (Table 2). The procedure is described in the following section

### At the beginning of a set of measurements:

- Select a range of 3 mA on the zero point galvanometer.
- Set up the smallest reference resistor.
- Position the sliding contact at about  $s_1 \approx 90 \text{ cm}$ .
- Turn on the power supply and set the voltage to 5 V.

### Recording measurements:

- Move the sliding contact to the position where the zero point galvanometer can no longer detect any current across it (balancing of the resistance bridge).
- Select a 30 μA range on the zero point galvanometer and carry out the balancing procedure as accurately as possible.
- Read off the length of the first part of the resistance wire  $s_1$  from the scale on the rail with the help of the pointer on the sliding contact. Enter the value into tables 1 and 2.
- Select the 30 mA range on the zero point galvanometer.
- Step by step, set up the next highest reference resistance and balance the bridge to zero for each one as described above. In each step, make sure that the galvanometer is not overloaded. You may need to move the sliding contact to roughly the right place in advance.

## SAMPLE MEASUREMENT AND EVALUATION

Table 1: Measurement of resistance for  $R_x = 100 \Omega$ . Set resistance values  $R_{ref}$ , lengths measured  $s_1$  and calculated resistance values with measurement errors determined in accordance with equation (4).

| $R_x \pm \Delta R_x / \Omega$<br>Nominal value | $R_{ref} \pm \Delta R_{ref} / \Omega$ | $s_1 \pm \Delta s_1 / \text{cm}$ | $R_x \pm \Delta R_x / \Omega$<br>Result |
|--|---------------------------------------|----------------------------------|---|
| 100±1  | 10.00±0.05                            | 91.00±0.05                       | 101.1±0.8                               |
|  | 50.00±0.25                            | 66.80±0.05                       | 100.6±0.6                               |
|  | 100.0±0.5                             | 50.10±0.05                       | 100.4±0.5                               |
|  | 500.0±2.5                             | 16.80±0.05                       | 101.0±0.6                               |
|  | 1000±5                                | 9.15±0.05                        | 100.7±0.8                               |

Table 2: Measurement of resistance for  $R_x = 1 \text{ k}\Omega$ . Set resistance values  $R_{ref}$ , lengths measured  $s_1$  and calculated resistance values with measurement errors determined in accordance with equation (4).

| $R_x \pm \Delta R_x / \Omega$<br>Nominal value | $R_{ref} \pm \Delta R_{ref} / \Omega$ | $s_1 \pm \Delta s_1 / \text{cm}$ | $R_x \pm \Delta R_x / \Omega$<br>Result |
|--|---------------------------------------|----------------------------------|---|
| 1000±10  | 100.0±0.5                             | 91.00±0.05                       | 1011±8                                  |
|  | 500.0±2.5                             | 66.75±0.05                       | 1004±6                                  |
|  | 1000±5                                | 50.05±0.05                       | 1002±5                                  |
|  | 5000±25                               | 16.75±0.05                       | 1006±6                                  |
|  | 10000±50                              | 9.15±0.05                        | 1007±8                                  |

- From the lengths  $s_1$  (Table 1, Table 2) measured, use equation (3) to determine the values  $R_x$  using various different reference resistances  $R_{ref}$  and use equation (4) to calculate the measurement error  $\Delta R_x$ . For each measurement, enter the results into Table 1.
- Taking into account the measurement error calculated for the values of  $R_x$  determined by experiment with the various reference resistances  $R_{ref}$  and lengths  $s_1$ , compare the values determined with the nominal values.

### Conclusion:

The values measured are in good agreement with the nominal values within the measurement errors determined for all reference resistance values and positions of the sliding contact. The error in the measurement is at its minimum when  $s_1 = s_2 = 50 \text{ cm}$  but does not vary significantly within the range  $10 \text{ cm} \leq s_1 \leq 90 \text{ cm}$  (cf. Fig. 3).

## OTHER MEASURING METHODS

### **Balancing by adapting the reference resistances**

- Position the sliding contact in the centre of the resistance wire such that  $s_1 = s_2 = 50$  cm.
- Set up the resistor decades in such a way that their reference resistance causes the needle of the zero point galvanometer to be as close as possible to the zero point.
- Move the sliding contact to the position where the needle of the galvanometer is exactly at the zero point and determine the resistance to be measured from the result.

### **Fixed reference resistance for various resistances to be measured**

- Swap over the precision resistors and the resistor decades so that the precision resistors now act as a fixed reference resistance  $R_{\text{ref}}$  and the resistor decades can be used to measure a variety of different resistance values  $R_x$ .
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