

Foucault Pendulum

DEMONSTRATE THE ROTATION OF THE EARTH WITH A FOUCAULT PENDULUM

- Measure the direction of the oscillation as a function of time.
- Determine the speed of the rotation.
- Determine the latitude where the experiment is taking place.

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Fig. 1 Foucault pendulum

GENERAL PRINCIPLES

A Foucault pendulum is a long string pendulum with a heavy bob, which can be used to demonstrate the rotation of the earth. It is named after *Jean Foucault*, who in 1851 discovered that the direction of the oscillation of a pendulum 2 m in length would change over the course of time. This experiment was later repeated with ever longer and heavier pendulums.

Since the earth rotates on its axis, when using an earth-based coordinate system, a force called the Coriolis force arises, which then acts on the moving pendulum in a direction perpendicular to the direction of the oscillation:

$$(F = 2 \cdot m \cdot \Omega_0 \times v) \quad (1)$$

m : Mass of pendulum bob

Ω_0 : Vector describing angular velocity of earth

v : Velocity vector of oscillating pendulum

This causes the plane of the oscillation to turn with an angular frequency dependent on the angle of latitude φ of the point from which the pendulum is suspended.

Because the Foucault pendulum is only deflected by a small angle α , the pendulum bob can be assumed to move exclusively in the horizontal plane, which can be seen in Fig. 2, and moves between an axis N aligned with north and an axis E aligned with east.

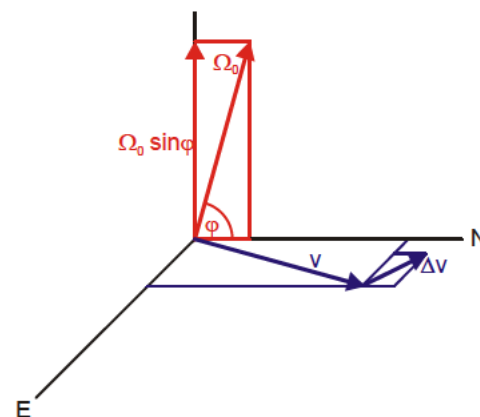


Fig. 2 Illustration of Foucault pendulum in fixed earth-based coordinate system.

The observation is concerned only with horizontal deflections since the pendulum bob is hanging from a thread. For this reason, only the vertical component of the vector Ω_0 is relevant:

$$\Omega(\varphi) = \Omega_0 \cdot \sin\varphi \tag{2}$$

The equation of motion for an oscillating Foucault pendulum is therefore as follows:

$$\frac{d^2\alpha}{dt^2} \cdot \mathbf{e}_\varphi + 2 \cdot \Omega_0 \cdot \sin\varphi \cdot \frac{d\alpha}{dt} \cdot \mathbf{e}_\varphi + \frac{g}{L} \cdot \alpha \cdot \mathbf{e}_\varphi = 0 \tag{3}$$

L : length of pendulum

g : acceleration due to gravity

\mathbf{e}_φ : horizontal unit vector parallel to the current direction of oscillation

\mathbf{e}_\perp : horizontal unit vector perpendicular to current direction of oscillation

The solution to this can be separated into a solution for the angle of deflection α and a solution for the turning unit vector \mathbf{e}_φ parallel to the current direction of oscillation:

$$\alpha(t) = \cos(\omega \cdot t + \beta) \text{ where } \omega = \sqrt{\frac{g}{L}} \tag{4a}$$

$$\mathbf{e}_\varphi(t) = \mathbf{e}_E \cdot \cos(\psi(t)) + \mathbf{e}_N \cdot \sin(\psi(t)) \tag{4b}$$

where $\psi(t) = \Omega_0 \cdot \sin\varphi \cdot t + \psi_0$: direction of oscillation

\mathbf{e}_E : horizontal unit vector aligned with east

\mathbf{e}_N : horizontal unit vector aligned with north

The plane of the oscillation therefore rotates over the course of time with a frequency as given by equation (2). In the northern hemisphere the rotation is clockwise and in the southern hemisphere it is anti-clockwise. The speed of the rotation is at its highest at the poles, whereas at the equator there is no rotation at all.

In this experiment, a pendulum 1.2 metre in length is used. In order to avoid the oscillations becoming elliptical, the pendulum thread is allowed to collide with a so-called Charon ring every time it swings. The direction of oscillation can be seen by projecting the shadow of the thread onto an angle scale whereby the angle can be read off with great accuracy. It is possible to observe the rotation of the plane of oscillation after only a few minutes. For long periods of observation, any damping of the oscillation can be compensated for with the aid of an adjustable electromagnetic system to provide additional momentum.

LIST OF EQUIPMENT

1	Foucault Pendulum @230 V	1000748 (U8403000-230)
or		
1	Foucault Pendulum @115 V	1000747 (U8403000-115)
1	Digital Stopwatch	1002811 (U11902)

SET UP

See instructions for use of the Foucault pendulum.

Choice of where to set up:

- Put the Foucault pendulum on a horizontal surface which is stable and will not shake.
- Avoid direct sunlight.

Checking the distance between the pendulum bob and the electro-magnet:

- Place the adjustment disc onto the measuring equipment and set up the length of the pendulum in such a way that the pendulum bob just touches the adjustment disc.
- Check this distance at lengthier intervals, since the pendulum string can extend by 1 to 2 mm.

Checking horizontal alignment:

- Place the adjustment cylinder onto the measuring equipment and suspend the pendulum from it.
- By turning the adjustable feet (two feet at once) align the Foucault pendulum such that the pendulum bob hangs down precisely in the middle of the adjustment cylinder.

Twisting of the pendulum thread:

- Allow the pendulum bob to hang freely for several hours in order to eliminate any twists in the thread.

EXPERIMENT PROCEDURE

- Deflect the pendulum with your hand and then release it.
- Carefully close the glass door.
- Turn on the electro-magnetic excitation system and observe the pendulum for at least 5 minutes.
- Set up the excitation system such that the pendulum does not stop and but also not collide with the glass door.
- Set up the angle of the light source for projecting a shadow in such a way that the shadow of the oscillating pendulum remains within the vertical markings on the observation screen.

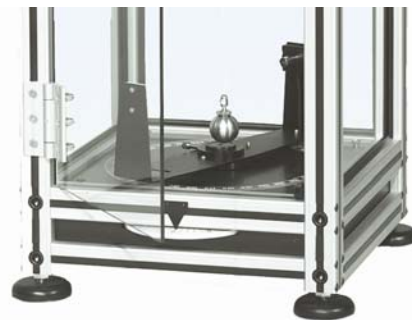


Fig. 3 Light source for projecting a shadow, observation screen and angle measurement disc for Foucault pendulum

- Read off the angle from the angle measurement disc and start measuring the time.
- Readjust the angle of the light source every 10 minutes or so such that the shadow of the pendulum continues to be within the vertical markings.
- Write down the angle and the measured time interval.

SAMPLE MEASUREMENT

Table 1: Measurements of plane of oscillation ψ and time t , recorded at geographical latitude $\varphi = 50^\circ$

t/s	ψ
0.0	179.6°
662.4	181.6°
1200.0	183.2°
1833.6	185.0°
3024.0	188.0°
3660.0	190.0°
4260.0	192.2°
5178.0	195.2°
5820.0	197.4°

EVALUATION

The angle of the oscillation plane ψ is in linear proportion to the time, see Fig. 4. The value we are seeking $\Omega(\varphi)$ is the gradient of the straight lines through the measurements.

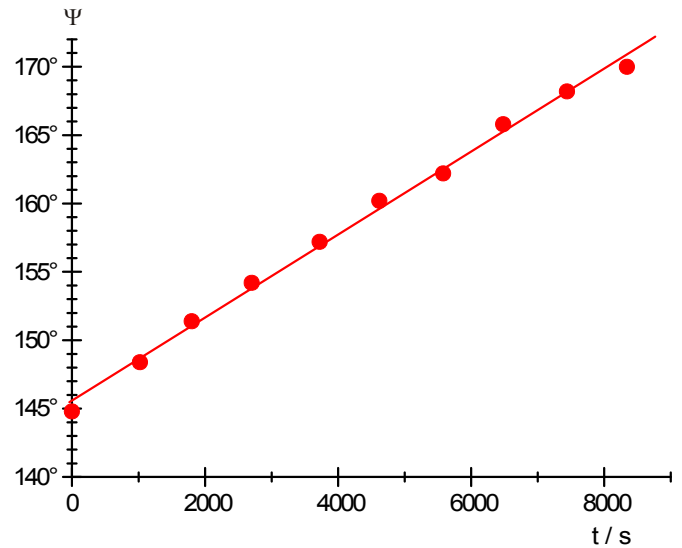


Fig. 4 Plot of angle ψ of the plane of oscillation as a function of time t , as recorded at latitude $\varphi = 50^\circ$

From the measurements obtained, the following can be derived

$$\Omega(\varphi) = (0.0030 \pm 0.0003) \text{ }^\circ/\text{s}$$

Rearranging Equation (2), the geographical latitude can be calculated using the following expression:

$$\varphi = \frac{180^\circ}{\pi} \cdot \arcsin\left(\frac{86400 \text{ s}}{360^\circ} \cdot \Omega(\varphi)\right) = 46^\circ \pm 4^\circ$$