PHYSICS EXPERIMENT

Rotational motion with uniform acceleration

CONFIRM NEWTON'S EQUATION OF MOTION.

- Plot the angle of rotation point by point as a function of time for a uniformly accelerated rotational motion.
- Confirm the proportionality between the angle of rotation and the square of the time.
- Determine the angular acceleration as a function of the torque and confirm agreement with Newton's equation of motion.
- Determine the angular acceleration as a function of the moment of inertia and confirm agreement with Newton's equation of motion.

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BASIC PRINCIPLES

The rotation of a rigid body about a fixed axis can be described in a way that is analogous to a one-dimensional translational motion. The distance *s* is replaced by the angle of rotation φ, the linear velocity *v* by the angular velocity ω, the acceleration *a* by the angular acceleration α, the accelerating force *F* by the torque *M* acting on the rigid body, and the inertial mass *m* by the rigid body's moment of inertia *J* about the axis of rotation.

In analogy to Newton's law of motion for translational motion, the relationship between the torque (turning moment) *M* that is applied to a rigid body with a moment of inertia *J*, supported so that it can rotate, and the angular acceleration α is:

$$M = J \cdot \alpha \qquad (1)$$

If the applied torque is constant, the body undergoes a rotational motion with a constant rate of angular acceleration.

In the experiment, this behaviour is investigated by means of a rotating system that rests on an air-bearing and therefore has very little friction. The motion is started at the time $t_0 = 0$ with zero initial angular velocity $\omega = 0$, and in the time *t* it rotates through the angle

$$\varphi = \frac{1}{2} \cdot \alpha \cdot t^2 \qquad (2)$$

Fig. 1: Experiment set-up for the study of uniformly accelerated rotational motions

LIST OF APPARATUS

1 Rotating System on Air Bed @ 230 V 1000782 (U8405680-230)
or
1 Rotating System on Air Bed @ 115 V 1000781 (U8405680-115)

1 Laser Reflection Sensor 1001034 (U8533380)

1 Digital Counter @ 230 V 1001033 (U8533341-230)
or
1 Digital Counter @ 115 V 1001032 (U8533341-115)

The torque M results from the weight of an accelerating mass m_M acting at the distance r_M from the axis of rotation of the body, and is therefore:

$$M = r_M \cdot m_M \cdot g \quad (3)$$

$g = 9.81 \frac{m}{s^2}$: the gravitational acceleration constant.

If two additional weights of mass m_J are attached to the horizontal rod of the rotating system at the same fixed distance r_J from the axis of rotation, the moment of inertia is increased to:

$$J = J_0 + 2 \cdot m_J \cdot r_J^2 \quad (4)$$

J_o: moment of inertia without additional weights.

A number of weights are provided, both for producing the accelerating force and for increasing the moment of inertia. The distances r_M and r_J can also be varied. Thus, it is possible to investigate how the angular acceleration depends on the torque and the moment of inertia in order to confirm the relationship (1).

SET-UP

- Set up the “rotating system on air bed” as described in the instruction sheet, and level it horizontally.
- Place on it the rotating disc with the horizontal rod and screw the multiple pulley on to that.
- Place the laser reflection sensor on the side-bracket of the start/stop unit.
- Move the release lever of the start/stop unit to its upper position.
- Switch on the air blower and move the start/stop unit so that its pointer touches the edge of the rotating disc and prevents it from turning freely.
- Rotate the disc until the pointer is at the zero (0°) position.
- Taking care to observe the colour-coding of the sockets, connect the start/stop unit to the “start” input of the digital counter and the laser reflection sensor to its “stop“ input.
- Position the laser reflection sensor so that the light beam passes through the hole at the 0° mark of the rotating disc.
- Move the function selector switch of the digital counter to the Δt_{AB} / ms position.

EXPERIMENT PROCEDURE

a) Point-by-point plotting of a uniformly accelerated rotational motion:

- Wind the thread around the middle groove of the multiple pulley (r_M = 10 mm) and over the other pulley, and hang on weights with a total mass of 3 g (m_M = 3 g).
- Rotate the disc to a starting angle (φ) of 10°.
- Start the rotation by pressing the lever and wait until the time measurement by the digital counter is completed.
- Read off the time t and enter the value in Table 1.
- Make similar time measurements with the angle φ set at 40°, 90° and 250°, and enter the results in Table 1.

b) Measure the angular acceleration as a function of the applied torque:

To determine the angular acceleration α as a function of the variables M and J, measure the time t(90°) needed for an angle of rotation of 90º with different values of the variable in both cases. For this special case the angular acceleration is

$$\alpha = \frac{\pi}{t(90°)^2}$$

- Rotate the disc to a starting angle of 90°.
- Hang a 1 g weight on the thread (mass m_M = 1 g).
- Start the rotation by pressing the release lever and wait until the time measurement by the digital counter is completed.
- Read off the time t(90°) and enter the value in Table 2a.
- Make similar time measurements with weights of mass m_M = 2 g, 3 g and 4 g and enter the results in Table 2a.
- From the time measurements calculate the angular accelerations α and enter the values in Table 2a.
- Wind the thread around the smallest of the multiple pulley wheels (r_M = 5 mm) and suspend weights with a total mass of 3 g (m_M = 3 g) from it.
- Again measure the time t(90°) for the disc to rotate by 90° and enter the value in Table 2b.
- Make a similar time measurement with pulley radius r_M = 15 mm and enter the result in Table 2b.
- From the time measurements calculate the angular accelerations α and enter the values in Table 2b.

c) Measure the angular acceleration as a function of the moment of inertia:

- Wind the thread around the medium-sized wheel of the multiple pulley (r_M = 10 mm) and suspend weights with a total mass of 3 g (m_M = 3 g).
- Measure the time t(90°) for the disc to rotate by 90° and enter the value in Table 3.
- Suspend two additional masses m_J = 50 g symmetrically from the horizontal rod at a distance r_J = 30 mm.
- Measure the time t(90°) and enter the value in Table 3.
- Increase the distance r_J in steps of 20 mm, measure the time t(90°) in each case, and enter the results in Table 3.

SAMPLE MEASUREMENTS

a) Point-by-point plotting of a uniformly accelerated rotational motion:

Table 1: Angle of rotation φ and time *t* in a uniformly accelerated rotational motion

φ	*t* / ms	φ	*t* / ms
0°	0	90°	3078
10°	1025	160°	4132
40°	2038	250°	5184

b) Measure the angular acceleration as a function of the applied torque:

Tab. 2a: Angular acceleration α as a function of the applied torque *M* (calculated using Equation 3). Force applied at the same radius r_M = 10 mm in every case.

m_M / g	*M* / mN mm	*t*(90°) / s	α / rad/s²
1	98	5.2	0.12
2	196	3.8	0.22
3	294	3.1	0.33
4	392	2.6	0.46

Tab.2b: Angular acceleration α as a function of the applied torque *M* (calculated using Equation 3). Force applied by the same suspended weight m_M = 3 g in every case.

r_M / mm	*M* / mN mm	*t*(90°) / s	α / rad/s²
5	147	4.4	0.16
10	294	3.1	0.33
15	441	2.5	0.50

c) Measure the angular acceleration as a function of the moment of inertia:

Tab. 3: Angular acceleration α as a function of the moment of inertia *J* (calculated using Equation 4, with J_0 = 0.873 g m²).
Parameters: m_M = 3 g, r_M = 10 mm, m_J = 50 g

r_J / mm	*J* / g m²	J_{total} / g m²	*t*(90°) / s	α/ rad/s²
0	0.000	0.873	3.098	0.33
30	0.090	0.963	3.277	0.29
50	0.250	1.123	3.46	0.26
70	0.490	1.363	3.857	0.21
90	0.810	1.683	4.276	0.17
110	1.210	2.083	4.724	0.14
130	1.690	2.563	5.231	0.11
150	2.250	3.123	5.778	0.09
170	2.890	3.763	6.307	0.08
190	3.610	4.483	6.93	0.07
210	4.410	5.283	7.481	0.06

EVALUATION

a) Numerical analysis and graphical analysis of a uniformly accelerated rotational motion:

Method 1, numerical analysis:

Calculate the ratios of the times for angles of rotation φ_0 = 10°, φ_1 = 40°, φ_2 = 90° and φ_3 = 250°

$$\frac{t(4\cdot\varphi_0)}{t(\varphi_0)} = \frac{2038\ \text{ms}}{1025\ \text{ms}} = 2.0\,,\quad \frac{t(9\cdot\varphi_0)}{t(\varphi_0)} = \frac{3078\ \text{ms}}{1025\ \text{ms}} = 3.0\,,$$

$$\frac{t(16\cdot\varphi_0)}{t(\varphi_0)} = \frac{4132\ \text{ms}}{1025\ \text{ms}} = 4.0\,,\quad \frac{t(25\cdot\varphi_0)}{t(\varphi_0)} = \frac{5184\ \text{ms}}{1025\ \text{ms}} = 5.1$$

Within the limits of accuracy of the measurements, for angles of rotation in the ratios 1 : 4 : 9 : 16 : 25 the corresponding rotation times are in the ratio 1 : 2 : 3 : 4 : 5. Thus the angle of rotation is proportional to the square of the time: $\varphi \propto t^2$.

Method 2, graphical analysis:

The experimental data can be presented as a plot of angle against time. The points can be fitted to a parabola, showing that the angle of rotation φ is not a linear function of the time *t* (see Fig. 2):

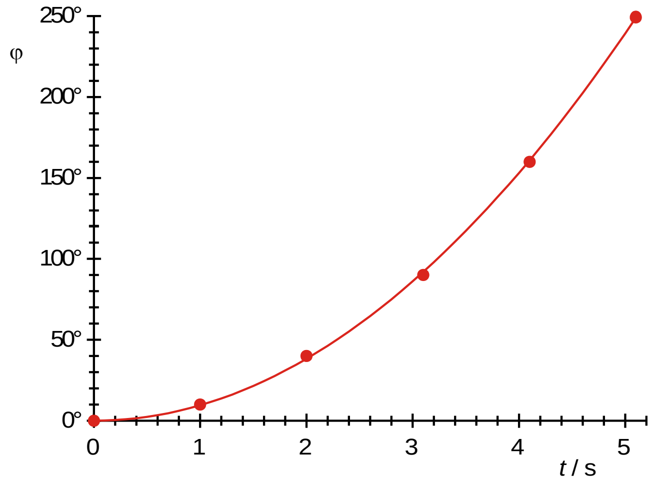

Fig. 2: Plot of angle against time for a uniformly accelerated circular motion

The relationship can be depicted as a straight by plotting the angle of rotation as a function of the square of the time (see Fig. 3):

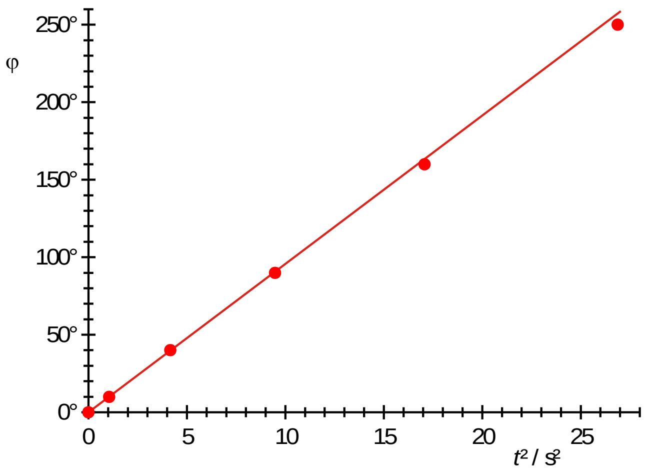

Fig. 3: Angle of rotation as a function of the square of the time

A straight line through the origin can be fitted to the experimental data, thus confirming Equation 2. The angular acceleration can be calculated from the gradient *A* of this line:

$$\alpha = 2 \cdot A = 18.68\,\frac{\text{grd}}{s^2} = 0.326\,\frac{\text{rad}}{s^2}$$

b) Measure the angular acceleration as a function of the applied torque:

Figure 4 presents the data from Tables 2a and 2b as a plot of angle of rotation against torque. Within the limit of accuracy of the measurements, they fit a straight line through the origin, thus confirming Equation 1.

$$\alpha = \frac{1}{0.873\ \text{g m}^2} \cdot M$$

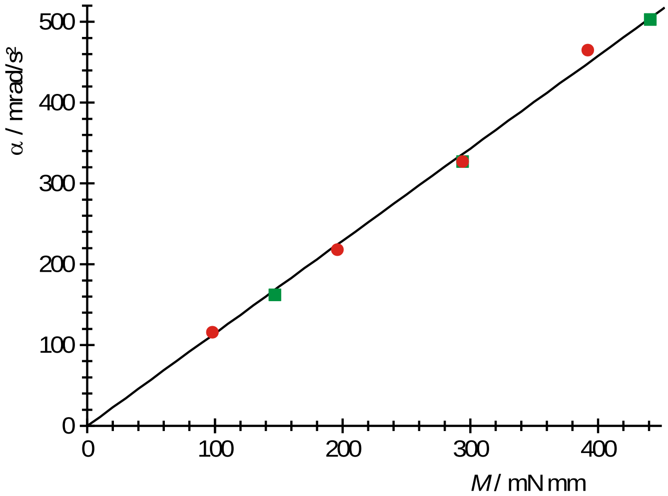

Fig. 4: Angular acceleration α as a function of applied torque *M* (● : r_M = 10 mm, ■ : m_M = 3 g)

c) Measure the angular acceleration as a function of the moment of inertia:

From the gradient of the straight line through the origin in Figure 4, the moment of inertia of the rotating disc with the horizontal rod alone is calculated as J_0 = 0.873 g m². This value has been used to calculate the total moment of inertia *J* in Table 3.

Figure 5 presents the data from Table 3 as a plot of angular acceleration against moment of inertia. Within the limitations of the accuracy of the measurements they fit a hyperbola drawn as shown, thus again confirming Equation 1.

$$\alpha = \frac{294\ \text{mN mm}}{\text{J}}$$

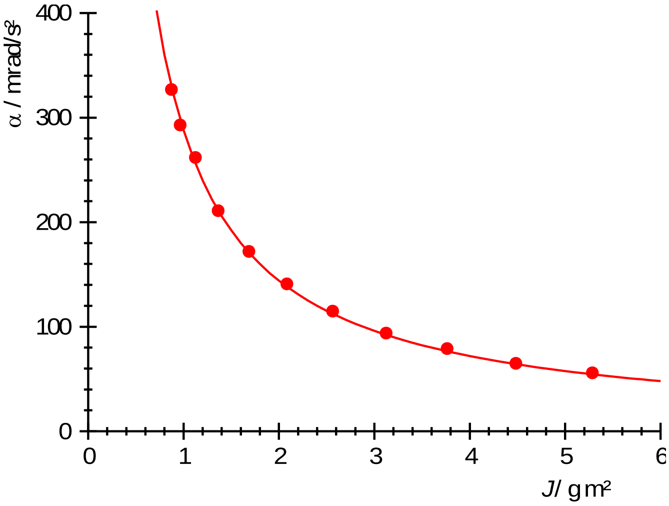

Fig. 5: Angular acceleration α as a function of the moment of inertia *J*.